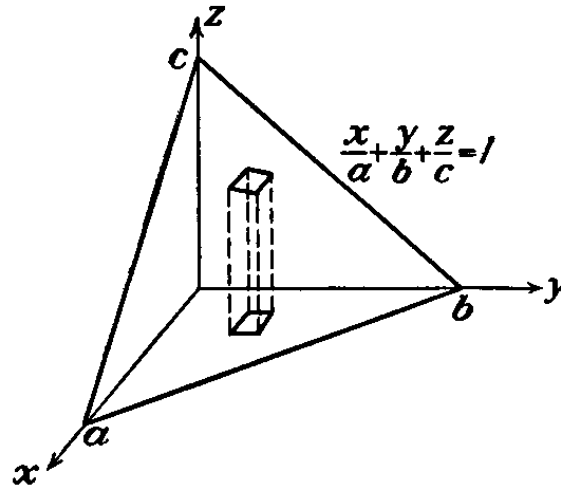


Calculus of several variables

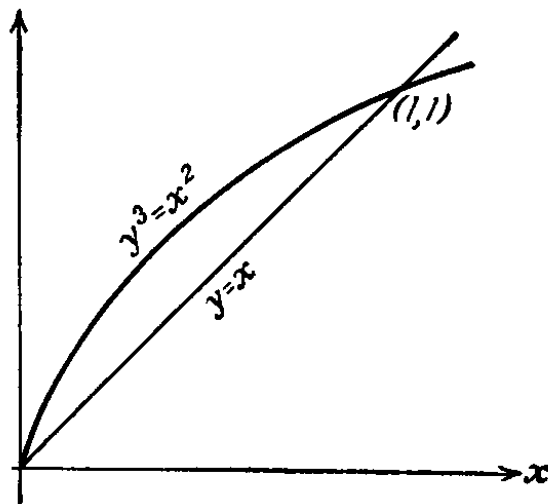
1. Find the volume of the tetrahedron bounded by the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ and the coordinate planes (fig.).



2. Evaluate using Green's theorem,

$$\int_C (x^2y \, dx + y^3 \, dy)$$

Where C is the closed path formed by $y = x$, and $y^3 = x^2$ from (0,0) to (1,1).



3. Show: $\vec{\nabla} \cdot (\vec{a} \times \vec{b}) = \vec{b} \cdot (\vec{\nabla} \times \vec{a}) - \vec{a} \cdot (\vec{\nabla} \times \vec{b})$

Calculus of Several Variables

Answer any two.

1. Find the x-coordinate of the center of gravity of the solid of uniform density ρ lying in the first octant and bounded by the three coordinate planes and the sphere $x^2 + y^2 + z^2 = r^2$. Use $dV = r^2 \sin\theta \, dr \, d\phi \, d\theta$.

2. Verify the divergence theorem for

$$\mathbf{A} = \mathbf{i} \frac{x}{r} + \mathbf{j} \frac{y}{r} + \mathbf{k} \frac{z}{r}$$

where $r^2 = x^2 + y^2 + z^2$ over the sphere with radius R.

3. (a) If $u = xy - yz$ and $x = r + s$, $y = r - s$, $z = t$, find $\partial u / \partial r$, $\partial u / \partial s$, and $\partial u / \partial t$.

(b) If $u = x^2 - y^2$ and $y = r \sin\theta$ and $x = r \cos\theta$, find $\partial u / \partial r$ and $\partial u / \partial \theta$.

Calculus of several variables

(Answer any two)

1. Are the following vector fields conservative?

(a) $F(x, y) = (2xy + 2x, x^2 - 6y)$

(b) $F(x, y) = \frac{1}{x^2+y^2}(-y, x)$

2. Evaluate using Green's theorem, the line integral $\int_C (x^5 + 3y) dx + (2x - e^{y^3}) dy$, where C is the circle centered at (1,5) of radius 2.

3. Calculate the flux of $\vec{F}(x, y) = x^2\hat{i} + (x + e^y)\hat{j} - \hat{k}$ over the rectangle $y = -1, 0 \leq x \leq 2, 0 \leq z \leq 4$ oriented in the negative y-direction.