The duration of this exam is 3 hours. There are 5 questions and the student must work 4 of them. If more than 4 questions are attempted, the student must specify in the box below which 4 are to be graded. If not, the first 4 questions will be graded.

Name: ___________________________________

Question	**Grade this one?**	**Student's grade**
1	25	
2	25	
3	25	
4	25	
5	25	
Total	100	

1. (a) (**12** *points*) What is the maximum degree of polynomials (n) for which the following Newton-Cotes formula can give the exact solution?

$$\int_0^3 f(x)\,dx \approx A_0 f(0) + A_1 f(1) + A_2 f(2) + A_3 f(3)$$

Determine the coefficients A_0, A_1, A_2, and A_3 so that the solutions are exact for all polynomials of degree $\leq n$.

(b) (**8** *points*) Using the <u>two-point</u> Gauss-Legendre quadrature to evaluate the following integration.

$$I = \int_{-1}^{1}\int_{-1}^{1}\left[cos(s)cos\left(\frac{t}{\sqrt{2}}\right) - \frac{s^2+t^2}{4000}\right]dsdt$$

No. of Points	**Location x_i**	**Weight Coefficient W_i**
1	$x_1 = 0.0$	$W_1 = 2.0$
2	$x_1 = -\sqrt{3}/3$ $x_2 = \sqrt{3}/3$	$W_1 = 1.0$ $W_2 = 1.0$
3	$x_1 = -\sqrt{0.6}$ $x_2 = 0$ $x_3 = \sqrt{0.6}$	$W_1 = 5/9$ $W_2 = 8/9$ $W_3 = 5/9$

(c) (**5** *points*) For $I = \int_{-1}^{1} f(x)\,dx$ where $f(x)$ is a polynomial, what is the maximum degree of $f(x)$ for which you would get the exact solution using the <u>four-point</u> Gauss-Legendre quadrature?

2. (**25** *points*) Let [**A**]{**x**} = {**b**} be the matrix form of the following system of equations:

$$
\begin{aligned}
80x_1 - 20x_2 - 20x_3 &= 20 \\
-20x_1 + 40x_2 - 20x_3 &= 20 \\
-20x_1 - 20x_2 + 130x_3 &= 20
\end{aligned}
$$

Obtain the Doolittle **LU** decomposition of the coefficient matrix [**A**]. Solve for x_1, x_2, and x_3 using *forward substitution* and *back substitution*. Show your work.

3. (a) (**5** *points*) Briefly discuss the advantages and disadvantages of direct and iterative methods for solving linear algebraic equations.

(b) (**5** *points*) What could be a potential problem of using Gaussian elimination to solve linear algebraic equations such as the one to the right? How would you resolve the issue?

$$\begin{bmatrix} 0.00001 & 2.0 & 3.0 \\ -1.0 & 3.712 & 4.623 \\ -2.0 & 1.072 & 5.643 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 1.0 \\ 2.0 \\ 3.0 \end{Bmatrix}$$

(c) (**5** *points*) What's the major differences between the Gauss-Seidel iteration method and Jacobi iteration method? Which one is considered to converge faster and why?

(d) (**10** *points*) Use Jacobi iteration method to solve for the given system of equations. Show the iterative equations for this problem, carry out three iterations, and put results in the following table.

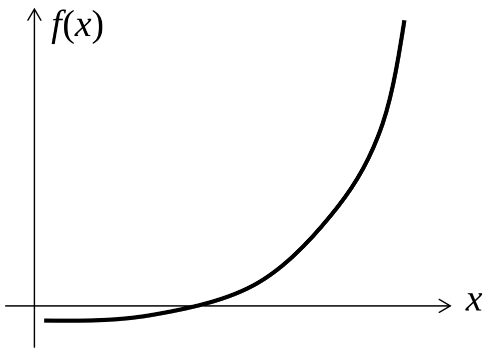
$$\begin{cases} 3x_1 + x_2 - x_3 = 6 \\ -x_1 + 2x_2 + 2x_3 = -5 \\ x_1 - x_2 + 4x_3 = 9 \end{cases}$$

k	x_1	x_2	x_3	R_1	R_2	R_3	Δx_{max}
0	0	0	0				–
1							
2							
3							

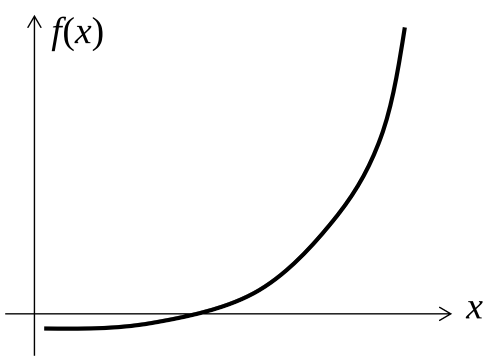
4. (a) (**5** *points*) Obtain an interpolation function for the data points in the table using the Lagrange interpolation method.

k	x_k	y_k
1	-2	-9
2	1	6
3	4	3

(b) (**10** *points*) Give a graphical description on how the root is found by Newton's method. Provide the algorithm (equations) for $xe^x - 2 = 0$ and obtain its root with an initial value of $x_0 = 1$. Use $\varepsilon = 0.001$ for convergence and show your work in a table.



(c) (**10** *points*) Give a graphical description on how the root is found by the Secant method. Provide the algorithm (equations) for $xe^x - 2 = 0$ and obtain its root with initial values of $x_0 = 1.2$ and $x_1 = 1$. Use $\varepsilon = 0.001$ for convergence and show your work in a table.



5. (a) (**5** *points*) What is isoparametric formulation? Why is it useful?

(b) (**20** *points*) Derive the stiffness matrix for the one-dimensional bar element shown in the figure using isoparametric formulation. The standard element in natural coordinates is shown in the bottom figure.