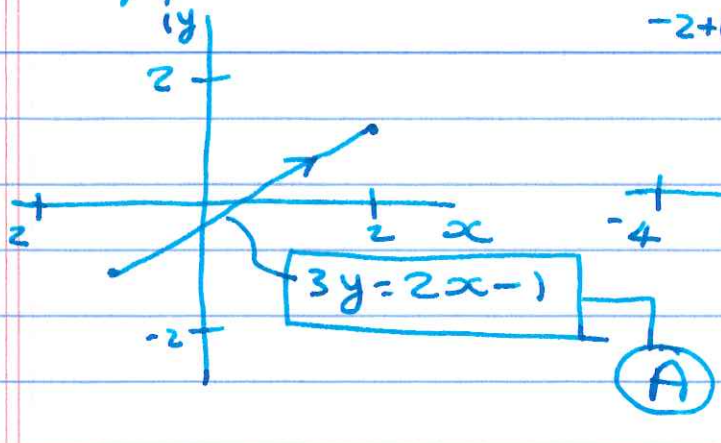
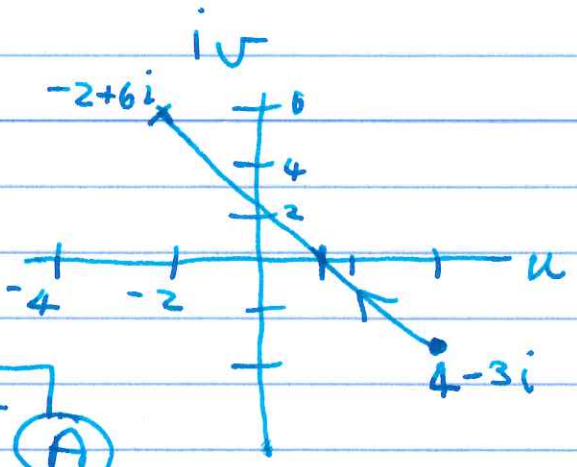
# PhD Qualifier Complex Variables Spring 2016

1 z-plane




a) $w = 3i(x+iy)+1 = 3ix - 3y + 1$

$u = -3y+1 \quad v = 3x$ (Straight line

b) $w_{start} = -\frac{1}{2} + \frac{1}{2}i \quad w_{stop} = \frac{2}{5} - \frac{1}{5}i$

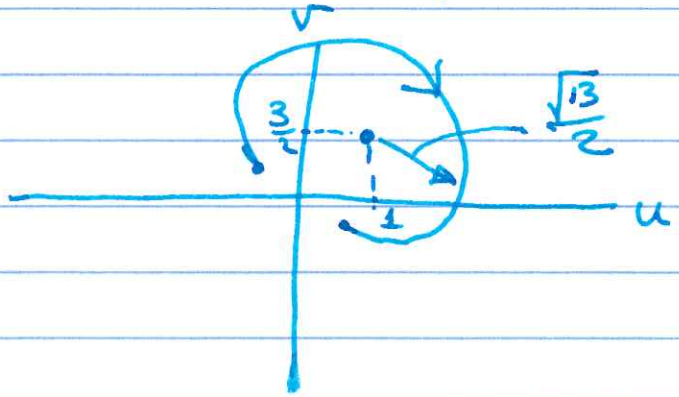
$$z = \frac{1}{w} = x + iy = \frac{u}{u^2+v^2} - i\frac{v}{u^2+v^2}$$

$(x) \qquad (y)$

B

B into A yields

$$(u-1)^2 + \left(v - \frac{3}{2}\right)^2 = \frac{13}{4}$$ A circle!



2. CR condition

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad \& \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

a) $\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = e^x \sin y + e^x - \sin y = 0$
$\therefore$ is harmonic

b) $2 - 4 \neq 0$ $\therefore$ is not harmonic

Now $\frac{\partial v}{\partial y} = e^x \cos y = \frac{\partial u}{\partial x}$ so

$$u = e^x \cos y + f(y)$$

$$\frac{-\partial v}{\partial x} = -e^x \sin y = \frac{\partial u}{\partial y} = -e^x \sin y + \frac{\partial f(y)}{\partial y}$$

$\therefore f(y) = c$

$$\boxed{u = e^x \cos y + c}$$

$$w = e^x \cos y + c + i e^x \sin y$$
$$= e^x (\cos y + i \sin y) + c$$
$$= e^z + c \qquad \text{(QED)}$$