

## *Probability and Statistics*

Sample problems similar to those on previous exams – issued Spring 2017

---

Two discrete random variables X and Y have the following probability distributions:

Variable X	
$x$	$p(x)$
1	0.3
2	0.2
3	0.1
4	0.4

Variable Y	
$y$	$p(y)$
1	0.1
2	0.5
3	0.2
4	0.2

1a) Calculate the probability that the quantity  $X + Y$  is greater than or equal to 6.

1b) Calculate the mean and variance for the random variable X (find  $\mu_x$ , and  $\sigma_x^2$ ).

---

2) Random variables A and B have the following mean and variance:

$\mu_A = 11$
$\sigma_A^2 = 4$
$\mu_B = 3$
$\sigma_B^2 = 1$

Calculate the mean and variance for a variable Z, where

a)  $Z = 3A - B$ .

b)  $Z = B^2 + 3AB$

---

We measure the length of 16 parts, and obtain the following data regarding the population of lengths:

Sample average: 105 mm

Sample standard deviation: 2.4 mm

3) Can we say that the true mean length of the total population of parts is greater than 103.7 mm at a 5% level of significance ( $\alpha$ )? *Clearly state the Null and Alternative Hypotheses, and your conclusion.*

---

4) Before going on vacation for a week, you ask your friend to water your sick plant. Without water, the plant has a 90 percent chance of dying. Even with proper watering, it has a 20 percent chance of dying. And the probability that your friend will forget to water it is 30 percent.

(a) What's the chance that your plant will survive the week?

(b) If it's dead when you return, what's the chance that your friend forgot to water it?

(c) If your friend forgot to water it, what's the chance it'll be dead when you return?

- 5) On the suspicion that a gaming dice does not produce a uniform chance of showing either of 6 numbers a series of sixty dice throws were used to produce the table of outcomes shown below.

<i>Outcome</i>	<i>Number of events</i>
1	18
2	7
3	5
4	11
5	7
6	12

At a 5% significance level, could this be considered to be a reasonable die?

---

- 6) State the integral equation for computing the mean and variance based on a knowledge of the probability density function  $p(x)$ .

Given that the probability density function  $p(x)$  for an exponentially random variable  $x$  is given by

$$p(x) = \lambda e^{-\lambda x} \quad x \geq 0$$

Determine the following:

- The cumulative distribution function  $F(x) = P(X \geq x)$ .
  - The mean.
  - The variance.
  - Assuming that a car battery wears out on average every 10,000 miles and its lifespan follows an exponential distribution, what is the probability that a battery will survive a 5000 mile trip?
- 

- 7) A manufacturing process produces parts that have a 98% probability of conforming with their specifications. A sample of 60 parts is taken. What is the probability that 2 or more parts in the sample of 60 will not conform with the specifications?
- 

- 8) A box of 100 parts has 6 that are defective. If you take 10 parts from the box, what is the probability that 1 or fewer of the 10 are defective?