Controls Qualifying Exam
Sample

Instructions: Complete the following five problems worth 20 points each. No material other than a calculator and pen/pencil can be used in the exam. A passing grade is approximately 70 points. If you do not understand something, make reasonable assumptions and state them clearly. This will be considered in the grading.

Laplace Transform Tables:

TABLE 2–1 Laplace Transform Pairs

	$f(t)$	$F(s)$
1	Unit impulse $\delta(t)$	1
2	Unit step $1(t)$	$\frac{1}{s}$
3	t	$\frac{1}{s^2}$
4	$\frac{t^{n-1}}{(n-1)!} \quad (n = 1, 2, 3, \ldots)$	$\frac{1}{s^n}$
5	$t^n \quad (n = 1, 2, 3, \ldots)$	$\frac{n!}{s^{n+1}}$
6	e^{-at}	$\frac{1}{s+a}$
7	te^{-at}	$\frac{1}{(s+a)^2}$
8	$\frac{1}{(n-1)!} t^{n-1} e^{-at} \quad (n = 1, 2, 3, \ldots)$	$\frac{1}{(s+a)^n}$
9	$t^n e^{-at} \quad (n = 1, 2, 3, \ldots)$	$\frac{n!}{(s+a)^{n+1}}$
10	$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
11	$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
12	$\sinh \omega t$	$\frac{\omega}{s^2-\omega^2}$
13	$\cosh \omega t$	$\frac{s}{s^2-\omega^2}$
14	$\frac{1}{a}(1 - e^{-at})$	$\frac{1}{s(s+a)}$
15	$\frac{1}{b-a}(e^{-at} - e^{-bt})$	$\frac{1}{(s+a)(s+b)}$
16	$\frac{1}{b-a}(be^{-bt} - ae^{-at})$	$\frac{s}{(s+a)(s+b)}$
17	$\frac{1}{ab}\left[1 + \frac{1}{a-b}(be^{-at} - ae^{-bt})\right]$	$\frac{1}{s(s+a)(s+b)}$
18	$\frac{1}{a^2}(1 - e^{-at} - ate^{-at})$	$\frac{1}{s(s+a)^2}$
19	$\frac{1}{a^2}(at - 1 + e^{-at})$	$\frac{1}{s^2(s+a)}$
20	$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
21	$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$

Problem 1: Figure 1 shows the block diagram for a control system whose objective is to make an output signal, y, track a human operator's joystick command, denoted by y_{des}. In order to filter out high frequency noise and vibrations from the human operator, the human input (y_{des}) is passed through a first order filter (known as a "precompensator") before being compared with y and sent to the main PI controller. The control signal (output of the main PI controller) is denoted by u, and the plant is represented by a second order transfer function as shown. Both the precompensator filter computations and PI controller computations are performed on a Renesas microcontroller.

a) Identify all of the components (blocks) in Figure 1 that represent computations that occur on the microcontroller (simply draw a dashed line around the set of these blocks).
b) How many sensors are required to implement the proposed control system? Which particular signals do these sensors need to measure?
c) In practice, control systems that are designed in the Laplace domain must ultimately be *realized* in the time domain. Derive a time domain *integral* realization for each of the blocks that you identified in part (a). You may submit your answer in one of two ways:
   - *Option 1:* Construct block diagram representations of each of the blocks you identified in (a), where the only elements in the block diagrams are gains, integrators, and summation junctions. *Your block diagram should not include derivatives!*
   - *Option 2:* Write closed-form, time domain expressions for $u(t)$ and $y_{des,f}(t)$. *Your closed-form expressions can include integrals but should not include derivatives!*

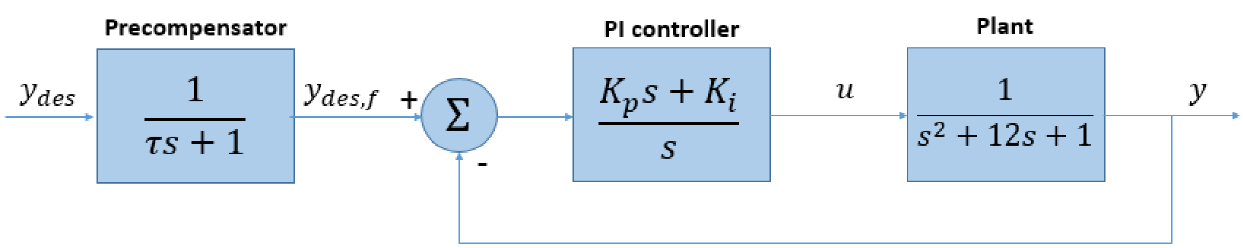

Figure 1: Control system (including plant) for problem 1.

Problem 2: Consider the feedback control system of Figure 2, where the forward path consists of a scalar gain, static nonlinearity, and *linear* mystery plant. The Bode plot of the mystery plant is shown in Figure 3, whereas the static nonlinearity is shown graphically in Figure 4.

What is the largest value of K for which the linearized closed-loop is stable for all linearization points where $-3 < y_0 < 3$ (y_0 represents the value of y around which the linearization is performed)? An approximate answer, based on your reading of the graphs, will be fine.

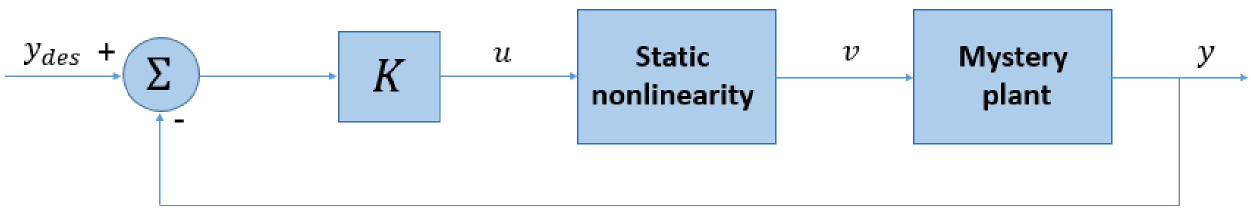

Figure 2: Block diagram for problem 2.

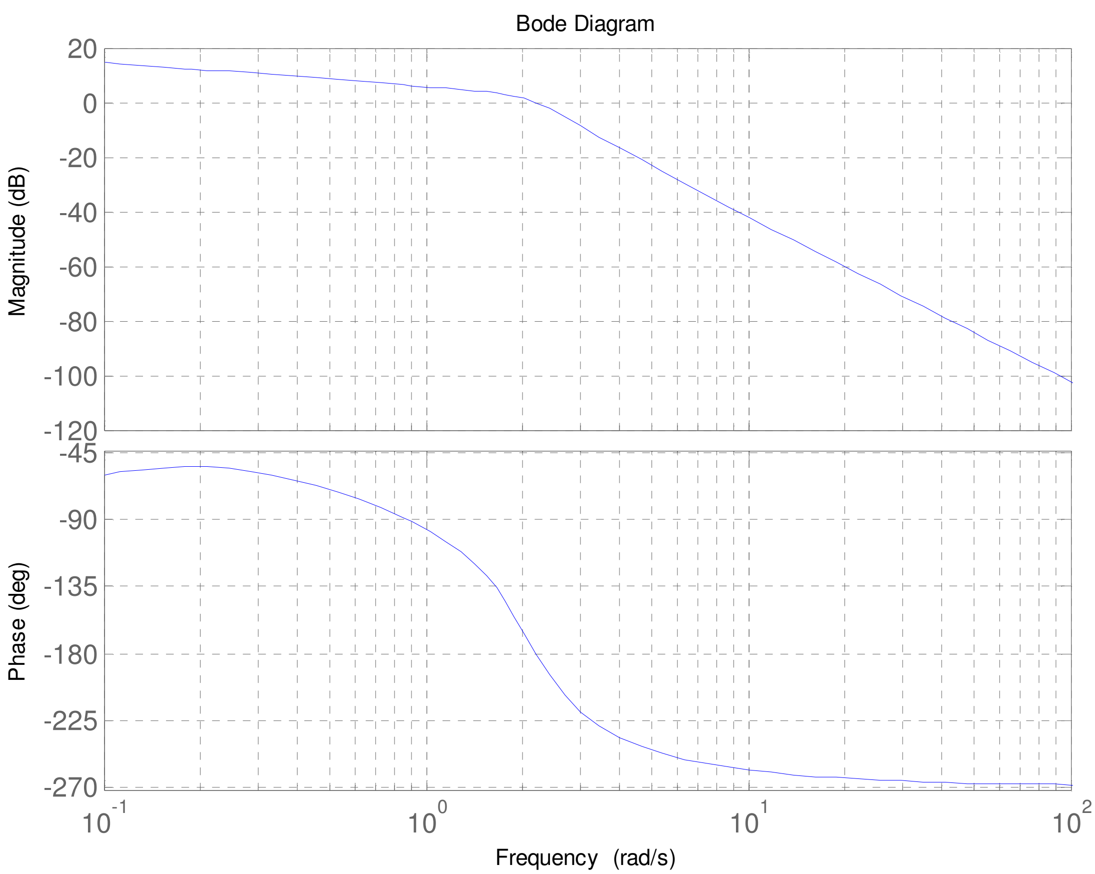

Figure 3: Bode plot for problem 2.

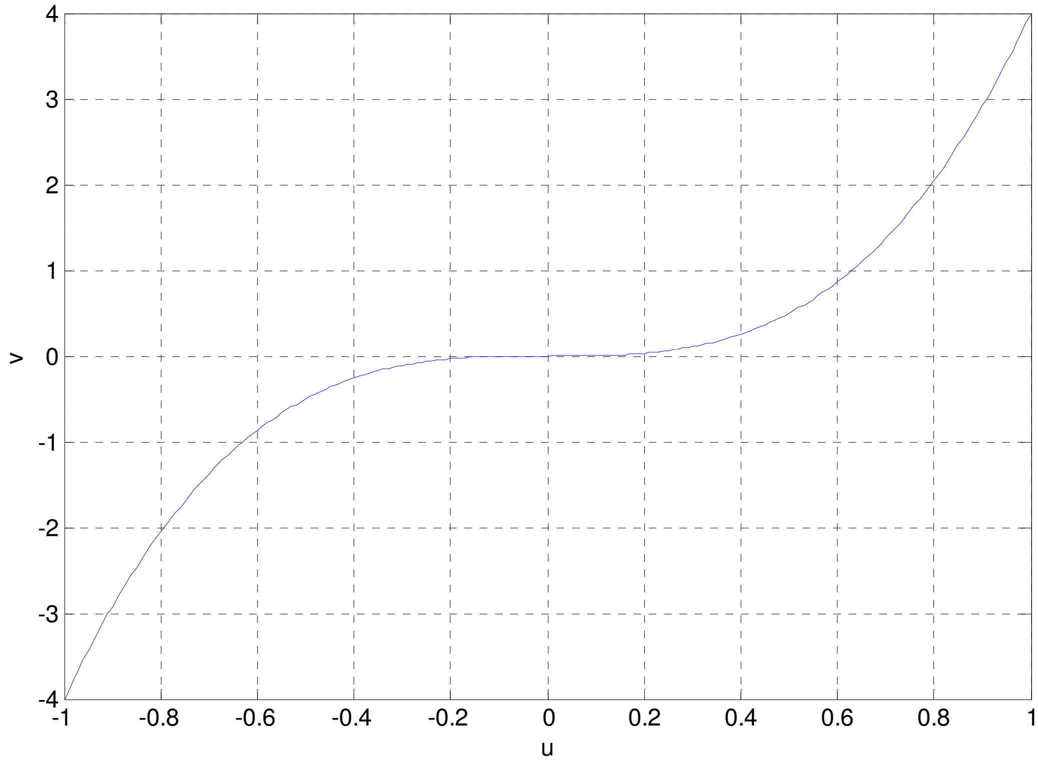

Figure 4: Static nonlinearity for problem 2.

Problem 3: Figure 5 shows a block diagram for a feedback control system, where a DC motor is used to control the position of a robot through an applied voltage, V. The motor is approximated with a first order transfer function in Figure 1. The control objective is to get the output position, y, to track the setpoint, y_{des}. v represents the velocity of the robot. The controller is a filtered proportional plus derivative controller, also known as a *lead filter*.

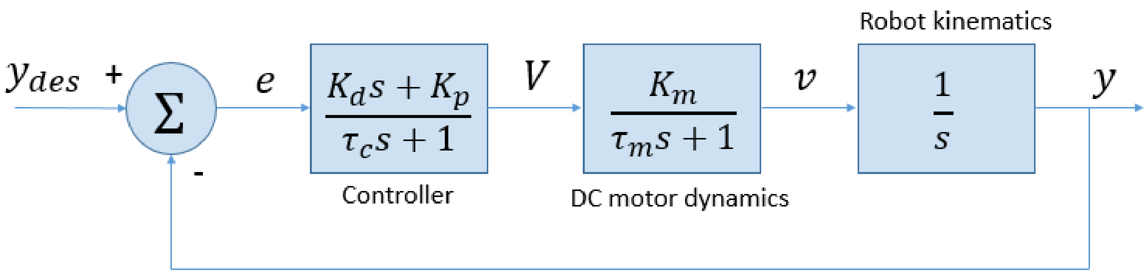

Figure 5: Block diagram for Problem 3.

(a) Derive the transfer function from y_{des} to y (i.e., derive $\frac{Y(s)}{Y_{des}(s)}$) in terms of the symbols in the block diagram. ***For full credit, your final transfer function should be the ratio of a numerator polynomial to a denominator polynomial. If your transfer function includes more than one fraction bar, expect to lose a lot of points.***

(b) Suppose that we choose τ_c, K_p, and K_d all to be positive (K_m and τ_m will be positive by their nature, as they are the motor gain and time constant, respectively). Furthermore, suppose that we choose the control gains such that $K_d/K_p = \tau_m$. Under the aforementioned assumptions, ***prove*** that the closed-loop system is input-output stable from y_{des} to y. *Hint: Under the stated assumption, the numerator of the controller can be factored as* $K_p(\tau_m + 1)$.

(c) Suppose that that $y_{des}(t)$ is a unit step input ($u(t)$). Under the same assumptions as part (b), calculate the steady-state value of $e(t)$. If it makes you feel better, you may assume all initial conditions are equal to zero – however, they will have no impact on the final value of $e(t)$. *Note: You do not need to have successfully completed part (b) to complete part (c).*

Problem 4: Consider the following open loop transfer functions as defined in the block diagram of Figure 6.

a. $\dfrac{K}{s(s\tau_1+1)(s\tau_2+1)}$

b. $\dfrac{K(s\tau_0+1)}{s^2(s\tau_1+1)}, \tau_0 > \tau_1$

c. $\dfrac{K}{s^2(s\tau_1+1)}$

For each case, determine whether the system will be inherently stable, conditionally stable, or inherently unstable under closed loop control.

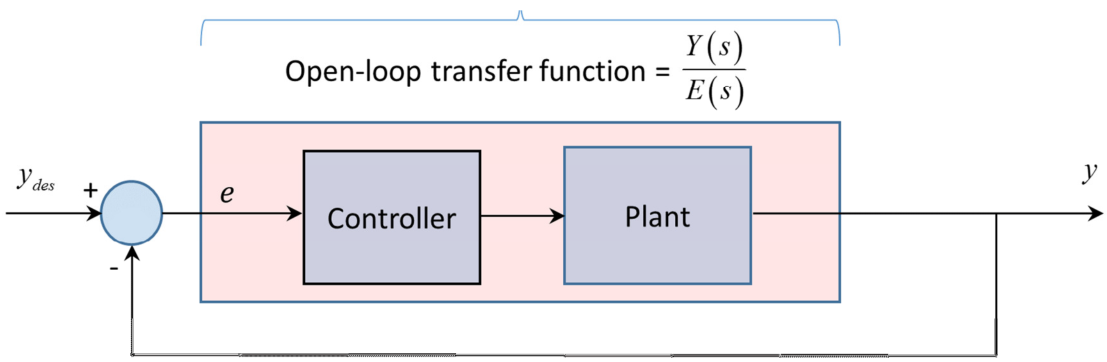

Figure 6: Open loop transfer function definition.

Problem 5: Consider the block diagram representation of a process controller shown in Figure 7 below.

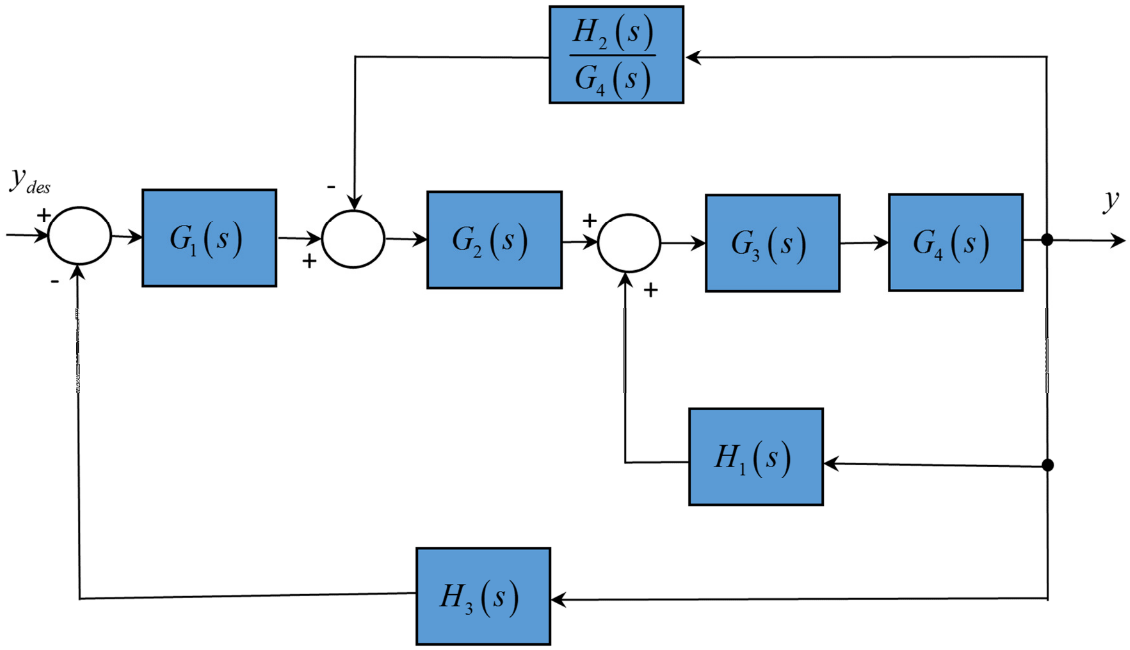

Figure 7: Block diagram of a process with three sensor feedback.

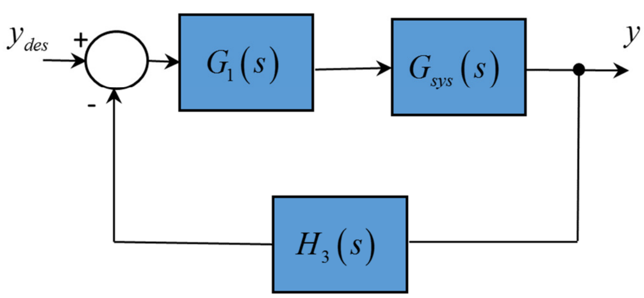

Figure 8: Equivalent block diagram to that shown in Figure 7.

Show that the block diagram in Figure 7 can be reduced to the following equivalent model shown in Figure 8 and determine the system transfer function $G_{sys}(s)$.