Ph.D. Qualifying Examination

Mechanics

Mechanical Engineering and Engineering Science Department
Fall 2009

Answer any four of the five questions posed. If all five problems are attempted, indicate in the table below which ones are to be graded. If there are no designated problems, the first four will be graded. Answer the elasticity question using the techniques of elasticity not mechanics of materials. Each problem is equally weighted.

Question	**Grade this one?**	**Grade**
1	Yes / No	
2	Yes / No	
3	Yes / No	
4	Yes / No	
5	Yes / No	
Total		

Table 1:

This is a closed book examination. I understand this and have not used any aids to help me in completing this examination. This examination represents my own work. I have not sought or been given any aid in completing this examination. I have followed the letter and spirit of the intent of the instruction for this examination.

____
Signature

____
Printed Name

Some Useful Equations:

Equations of Motion:

$$T_{ij,j} + \rho b_i = \rho \frac{\partial^2 u}{\partial t^2}$$

Equations of Equilibrium:

$$T_{ij,j} + \rho b_i = 0$$

Hooke's Law:

$$T_{ij} = \lambda \varepsilon_{kk} \delta_{ij} + 2\mu \varepsilon_{ij}$$

Inverse Relation:

$$\varepsilon_{ij} = \frac{1+\nu}{E} T_{ij} - \frac{\nu}{E} T_{kk} \delta_{ij}$$

Biharmonic Equation:

$$\frac{\partial^4 \phi}{\partial x_1^4} + 2\frac{\partial^4 \phi}{\partial x_1^2 \partial x_2^2} + \frac{\partial^4 \phi}{\partial x_2^4} = 0$$

Stress Components for Airy's Stress Function without Body Forces:

$$T_{11} = \frac{\partial^2 \phi}{\partial x_2^2}, \quad T_{12} = -\frac{\partial^2 \phi}{\partial x_1 \partial x_2}, \quad T_{22} = -\frac{\partial^2 \phi}{\partial x_1^2}$$

Mechanics of Materials

1. (a) A basketball is modelled as a thin-walled sphere. The internal pressure is p, the radius of the basketball is R, and the wall thickness is t. Derive an expression for the stress in the wall of the basketball if the wall thickness $t \ll R$ neglecting the stress in the radial direction of the basketball.

Why can the radial stress be neglected in this problem?

If the basketball is made of incompressible material the volume of the material is constant, $V_0 = V$. Use this information to express the stress as a function of the current radius only and known initial quantities.

(b) A rigid bar of weight W is suspended from three equally spaced cables. The outer two cables, S, are steel, and the inner cable, A, is aluminum. How must the areas of the cables be related if the force in each of the steel cables is twice that in the aluminum cable? Assume that $E_S = 3E_A$.

Mechanics of Materials

2. Determine the deflection curve for the beam shown in the figure below. Use the fourth order beam bending equation

$$\frac{d^2}{dx^2}\left(EI\frac{d^2v}{dx^2}\right) = -q(x)$$

Here v is the deflection, E is the modulus of elasticity, I is the moment of inertia of the cross section, and $q(x)$ is a distributed loading on the beam.

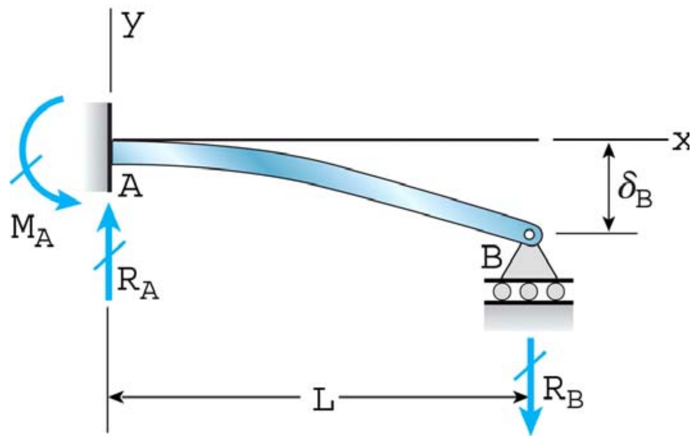

Figure 1:

Theory of Elasticity

3. (a) A particular rigid body deformation is described by

$$\mathbf{x} = \mathbf{c} + \mathbf{R}\mathbf{X}$$

where

$$[\mathbf{R}] = [R_{ij}] = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\mathbf{c}$ is a constant vector, and $\mathbf{x}$ and $\mathbf{X}$ are the final and initial positions of a particle. Compute the small strain tensor,

$$\boldsymbol{\varepsilon} = \frac{1}{2}\left[\text{grad}\,\mathbf{u} + (\text{grad}\,\mathbf{u})^{T}\right]$$

When will $\boldsymbol{\varepsilon}$ equal zero?

(b) Write the integral expressions for the balance of linear momentum and balance of moment of momentum (angular momentum) for a deformable body subject to surface tractions and body forces.

(c) Answer the following questions in short essays.
When is the stress tensor symmetric?
Why is the small-strain tensor symmetric, $\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}^{T}$

Theory of Elasticity

4. (a) In 1892, Beltrami proposed a general solution to the equilibrium equations for an elastic body without body forces

$$\mathbf{T} = \operatorname{curl}\operatorname{curl}\mathbf{A} \quad \text{or} \quad T_{ij} = \varepsilon_{imn}\varepsilon_{jpq}A_{mp,nq}$$

where $\mathbf{A} = \mathbf{A}^T$ is any sufficiently smooth symmetric second order tensor. Verify that this representation of the stress tensor satisfies the equilibrium equations.

(b) What are the boundary conditions for prescribed tractions and displacements that must be satisfied by this stress representation?

Theory of Elasticity

5. For the thin rectangular plate shown below, an Airy's stress function is

$$\phi = Ax_1^3 + Bx_1^2x_2 + Cx_1x_2^2 + Dx_2^3$$

Determine the stress components associated with this stress function. Discuss the problems that the stress function solves when

(a) $A = B = C = 0$ and $D \neq 0$

(b) $A = D = 0$ and B, C are not zero.

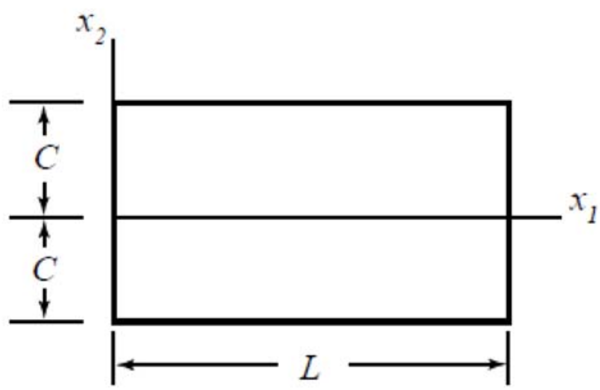

Figure 2: