Syllabus for the Mathematics Qualifying Exam

Department of Mechanical Engineering and Engineering Science

University of North Carolina at Charlotte

Students must choose four (4) of the following nine (9) mathematics topics:

Vector Algebra
Algebraic and geometric properties of vectors in three dimensions, dot and cross products, linear independence of vectors, scalar triple product, equations of lines and planes.

Linear Algebra
The linear algebra portion of the exam expects you to understand how matrices are used to represent systems of linear equations, various methods of matrix decomposition, the solution of matrix equations, eigenvectors and eigenvalues, diagonalization and canonical forms, orthogonalization techniques, kernel and image of matrices, linear interdependence, idempotency, projections (vector space), bases and dimension of vector spaces, positive definiteness, hermitian/unitary/similar matrix concepts, singular values, symmetric/skew-symmetric matrices, pseudo-inverse matrix, and other concepts based on determinant, trace, adjoint and rank of matrices.

Questions involving calculation may ask you (for example) to find the rank of a matrix, perform LU and QR matrix decomposition, find the solution of a matrix equation based on Cramer's rule or Gauss elimination, or determine the eigenvectors and eigenvalues of a matrix.

Conceptual questions may ask you (for example) about the relationship between eigenvalues and the SVD, or how the rank of a matrix product is dependent on the ranks of the matrices being multiplied, or why (ie. prove) the trace remains invariant under product operation on two matrices.

REFERENCE TEXTS: (1) Advanced Engineering Mathematics by Erwin Kreyszig (2) Linear Algebra by Kenneth Hoffman and Ray Kunze.

Calculus of One Variable
Functions, limits, continuity, differentiation: products, quotients, chain rule, intergration: by parts, by substitutions, chain rule, calculation of areas, volumes and arclengths.

Calculus of Several Variables
Scalar and vector fields, directional derivatives, gradient operator, divergence and curl of a vector, line, double and triple integrals, change of variables in integrals, integral theorems, conservative fields and potential functions.

Ordinary Differential Equations
Exact first-order equations, integrating factors, separable and homogeneous first-order equations, second-order equations reducible to first-order, solutions of nonhomogeneous equations, variation of parameters, second-order ordinary differential equations with constant coefficients (both homogeneous and nonhomogeneous equations), power-series solutions (the Frobenius method), systems of linear differential equations with constant coefficients.

Partial Differential Equations
Classification of linear partial differential equations, transient and steady-state problems in rectangular and cylindrical coordinate systems, boundary and initial conditions, separations of variables, eigenfunctions and orthogonality, eigenfunction expansions, Bessel functions, modified Bessel functions, the Fourier series, Laplace transform techniques for solving partial differential equations.

Numerical Methods
Decimal places and significant figures, roots of a single nonlinear equation: bisection method, Newton's method, interpolation: Newton's divided-difference interpolating polynomials and Lagrange polynomials, difference approximations to derivatives, numerical integration of integrals: trapezoidal rule, Simpson's rules and Gauss quadrature, numerical solution of linear systems of equations: direct methods based on the Gaussian elimination technique.

Complex Variables
Complex numbers and functions, geometric representation, poles and zeros of a complex function, roots of a polynomial, analytic functions, Taylor Series and Laurent Series, Cauchy-Riemann equations.

Probability and Statistics
Discrete and continuous random variables. Probability density functions and cumulative distribution functions. Finding the mean and variance of probability distributions and linear combinations of probability distributions. Finding the sample average and sample variance of experimental data. Hypothesis tests that use the Normal-, T-, Chi-square, and F-distributions.

Suggested books for study:

Students may find the following books useful for studying the math qualifying exam. However, please note that some of the topics above may not be covered by the listed references.

- Advanced Engineering Mathematics by E. Kreyszig, Wiley, 1998, ISBN: 0471154962
- Advanced Engineering Mathematics by Wylie and Barrett, McGraw-Hill, 1995, ISBN: 0070722064
- Advanced Engineering Mathematics by Alan Jeffrey, Harcourt Academic Press, 2002, ISBN: 012382592X
- Calculus 6/E, Edwards and Penney, Prentice Hall, 2002, ISBN: 0130920711
- Numerical Methods for Engineers, Chapra and Canale, McGraw-Hill, 2001, ISBN: 0072431938