1. Find the volume of the tetrahedron bounded by the plane \( \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1 \) and the coordinate planes (fig.).

2. Evaluate using Green’s theorem,

\[ \int_C (x^2y \, dx + y^3 \, dy) \]

Where C is the closed path formed by \( y = x \), and \( y^3 = x^2 \) from \((0,0)\) to \((1,1)\).

3. Show: \( \nabla \cdot (\vec{a} \times \vec{b}) = \vec{b} \cdot (\nabla \times \vec{a}) - \vec{a} \cdot (\nabla \times \vec{b}) \)
Calculus of Several Variables

Answer any two.

1. Find the x-coordinate of the center of gravity of the solid of uniform density $\rho$ lying in the first octant and bounded by the three coordinate planes and the sphere $x^2 + y^2 + z^2 = r^2$. Use $dV = r^2 \sin \theta \, dr \, d\phi \, d\theta$.

2. Verify the divergence theorem for

$$A = \mathbf{i} \frac{x}{r} + \mathbf{j} \frac{y}{r} + \mathbf{k} \frac{z}{r}$$

where $r^2 = x^2 + y^2 + z^2$ over the sphere with radius R.

3. (a) If $u = xy - yz$ and $x = r + s$, $y = r - s$, $z = t$, find $\frac{\partial u}{\partial r}$, $\frac{\partial u}{\partial s}$, and $\frac{\partial u}{\partial t}$.

(b) If $u = x^2 - y^2$ and $y = r \sin \theta$ and $x = r \cos \theta$, find $\frac{\partial u}{\partial r}$ and $\frac{\partial u}{\partial \theta}$. 
Calculus of several variables

(Answer any two)

1. Are the following vector fields conservative?
   
   (a) \( \mathbf{F}(x, y) = (2xy + 2x, x^2 - 6y) \)
   
   (b) \( \mathbf{F}(x, y) = \frac{1}{x^2 + y^2} (-y, x) \)

2. Evaluate using Green’s theorem, the line integral \( \int_C (x^5 + 3y) \, dx + (2x - e^{y^3}) \, dy \), where \( C \) is the circle centered at \((1, 5)\) of radius 2.

3. Calculate the flux of \( \mathbf{F}(x, y) = x^2 \mathbf{i} + (x + e^y) \mathbf{j} - \mathbf{k} \) over the rectangle \( y = -1, 0 \leq x \leq 2, 0 \leq z \leq 4 \) oriented in the negative \( y \)-direction.