1. Find the intervals on which the following differential equations are normal:
   (a) \( y'' + 7xy' - 11y = \ln \sin \pi x \),
   (b) \( \sqrt{x(1 - x)}y''' - e^{-x} \sin xy' + y = 2 - x \),
   (c) \( x^2y''' - 3xy'' + 4y = \sinh x \).

2. Consider the differential equation
   \[ y'' + 4y = 4 \text{ on } (-\infty, \infty) \]
   with \( y(0) = 2 \) and \( y'(0) = 0 \). Given that both \( 1 + \cos 2x \) and \( 2 \cos^2 x \) satisfy this differential equations and the initial conditions, what argument would you use to conclude that the solutions are equal, i.e.,
   \[ 1 + \cos 2x = 2 \cos^2 x \].

3. Determine if the following set of functions are linearly dependent or independent. If they are linearly dependent, provide a relationship that shows the dependence.
   (a) \( \{ e^x, x, \cosh x \} \) on \( (-\infty, \infty) \),
   (b) \( \{ e^x, e^{2x} \} \) on \( (-\infty, \infty) \),
   (c) \( \{ x^2 - 1, x^2 + x + 1, x^2 + 3x + 5 \} \) on \( (-\infty, \infty) \).

4. During the lecture, we remarked on the superposition principle for the particular solutions of non-homogeneous linear ordinary differential equations. Use this principle to obtain a particular solution to
   \[ y'' - 6y' + 5y = -10x^2 - 6x + 32 + e^{2x} \]
given that \( 3e^{2x} \) and \( x^2 + 3x \) are respectively particular solutions of
   \[ y'' - 6y' + 5y = -9e^{2x} \] and \( y'' - 6y' + 5y = 5x^2 + 3x - 16 \).