Sample Exam: Controls

Instructions: Welcome to quals! First, put your name on the paper you turn in (ideally a Blue Book). There are 4 problems on this exam, each of which is worth 25 points. The exam is closed notes, closed book. Calculators may be used. Good luck!

Problem 1: Consider the combined feedforward/feedback control structure shown in Figure 1.

Suppose that the constituent transfer functions are given by:

\[ C_{ff}(s) = K_{ff}, \] (1)
\[ C_{fb}(s) = K_{fb}, \] (2)
\[ P(s) = \frac{1}{1.5s + 1}, \] (3)
\[ H(s) = \frac{1}{0.5s + 1}, \] (4)

where \( K_{ff} \) and \( K_{fb} \) are positive scalar values (gains).

a) (15 points) For what (positive) values of \( K_{ff} \) and \( K_{fb} \) is the closed-loop system stable? Your answer should indicate a range of \( K_{ff} \) and \( K_{fb} \) for which the system is stable. If you think the system is stable for all or no values of \( K_{ff} \) or \( K_{fb} \), you may just say “stable for all values” or “stable for no values.”

b) (10 points) Assuming \( K_{fb} = 8 \), what value of \( K_{ff} \) is required to achieve a DC gain of 1 from \( y_{des} \) to \( y \)?

Problem 2: The longitudinal dynamics of a luxury cruise ship (which are similar to the longitudinal dynamics of, say, a Carnival Cruise Lines non-luxury ship, except when the Carnival ship runs aground) are described by the following equations:

\[ \tau_{eng} \dot{F}_{th} = F_{cmd} - F_{th}, \] (5)
\[ m\dot{v} = F_{th} - F_{drag}, \] (6)
\[ F_{drag} = k_{aero}(v + v_{wind})^2 + k_{hydro}(v + v_{cur})^2, \] (7)

where the variables are defined as follows:

\( F_{th} \) = Thrust force (N)
\[ F_{cmd} = \text{Commanded thrust force (N)} \]
\[ \tau_{eng} = \text{Engine time constant (s)} \]
\[ m = \text{Ship mass (including “added mass” of displaced fluid) (kg)} \]
\[ F_{drag} = \text{Retarding force from aerodynamic and hydrodynamic drag (N)} \]
\[ v = \text{Ship speed (m/s)} \]
\[ v_{wind} = \text{Wind speed (m/s)} - \text{Positive denotes a headwind} \]
\[ v_{cur} = \text{Current speed (m/s)} - \text{Positive denotes current flowing against the ship} \]
\[ k_{aero} = \text{Lumped aerodynamic drag coefficient (N s}^2 \text{m}^{-2}) \]
\[ k_{hydro} = \text{Lumped hydrodynamic drag coefficient (N s}^2 \text{m}^{-2}) \]

a) (15 points) Derive a linear approximation of the aforementioned longitudinal dynamic equations, around the equilibrium point \( v_0 = 10 \text{ m/s}, v_{wind,0} = 5 \text{ m/s}, v_{cur,0} = 0 \text{ m/s}, F_{th,0} = 42,500 \text{N}, F_{cmd,0} = 42,500 \text{N}, \) where the constant parameters are given by:

\[ \tau_{eng} = 10 \text{ s} \]
\[ m = 4 \cdot 10^7 \text{ kg} \]
\[ k_{aero} = 100 \frac{Ns^2}{m^2} \]
\[ k_{hydro} = 200 \frac{Ns^2}{m^2} \]

b) (10 points) Derive the transfer functions from \( \delta F_{cmd}, \delta v_{wind}, \) and \( \delta v_{cur} \) to \( \delta v \) for the linearized model you derived in (a).

**Problem 3:** Consider the two control system configurations in Figure 2, where \( y_{des} \) represents a setpoint, \( d \) represents an external disturbance, and \( n \) represents sensor noise. For each of the following three sets of circumstances and control requirements, indicate which of the two control structures you would recommend. *Answers should be supported by mathematical analyses to receive full credit.*

a) (10 points) Suppose that the plant dynamics are not known precisely, but it is known that \( 0.5 \leq a \leq 1.5 \). Furthermore, assume that the closed-loop performance requirements are:

- Stable closed-loop system;
- DC (steady-state) gain of 1 from \( y_{des} \) to \( y \).

b) (10 points) Suppose that the plant dynamics are known precisely, with \( a = 1 \), and that the new closed-loop performance requirements are:

- Stable closed-loop system;
- DC (steady-state) gain of 1 from \( y_{des} \) to \( y \);
- DC (steady-state) gain of 0 from \( d \) to \( y \).
Figure 2: Block diagrams for the configurations to be analyzed in problem 3. Note: the negative sign by the summation junction indicates negative feedback.

c) (5 points) Suppose that the plant dynamics are known precisely, with $a = 1$, and that the new control requirements are as follows:

- Stable closed-loop system;
- DC (steady-state) gain of 1 from $y_{des}$ to $y$;
- DC (steady-state) gain of 0.5 or less from $-n$ to $y$.

Problem 4: Consider the block diagram of Fig. 3, where a scalar gain ($K$) and first order filter (with time constant $\tau$) are cascaded with a linear mystery system having a transfer function $G(s)$, whose Bode plot is given in Fig. 4.

![Figure 3: Block diagram for problem 4.](image)

a) (15 points) Suppose that $\tau = 0.00001$. What is the maximum gain, $K$, for which the closed-loop system is stable?

b) (10 points) Suppose that $K = 1$. As the filter time constant, $\tau$, is increased, does the system’s phase margin increase, decrease, or stay the same?
Figure 4: Bode plot of $G(s)$ for problem 4.